A Cautious Self-Tuning Controller for Chemical Processes

An adaptive controller is presented that is able to perform effectively in the presence of nonlinearities and of model uncertainty typical of industrial processes. The controller incorporates a minimum variance part and a caution part. The latter is independent of the estimated model and becomes dominant in the case of a large mismatch between plant and estimated model. Application to a simulated nonlinear continuous stirred-tank reactor demonstrates its effectiveness, even when other adaptive controllers give unsatisfactory performance.

A. V. Papadoulis, C. A. Tsiligiannis, S. A. Svoronos
Department of Chemical Engineering
University of Florida
Gainesville, FL 32611

Introduction

There has been a growing interest in the application of self-tuning controllers (STC's) to chemical processes. The main reasons for this are that:

- STC's do not require a detailed process model
- Their design is conceptually simple
- They can adapt to slow changes of the process

In using an STC one tries to circumvent the intermingled problems of modeling and controller design that require determination of a system structure capable of capturing the main dynamic features of the actual process, meanwhile being simple enough to allow for an explicit controller design.

Adaptive controllers are generally based on on-line identified linear models. The stability and convergence to the target steady state of several adaptive controllers, including the popular minimum-variance (MV) STC (Åström and Wittenmark, 1973), when applied to linear plants was recently established (e.g. Goodwin et al., 1984). However, most chemical processes are inherently nonlinear due to the nonlinear dependence of reaction rates and physical properties on temperature, concentration, pressure, and geometry. Other nonlinearities are added by nonlinear sensors and control valves as well as by controller saturation. The existing linear-model-based adaptive algorithms face several problems when applied to highly nonlinear systems. For instance, the MV STC may lead to poor behavior due to plant/model mismatch during initialization, large set point changes, or load changes. One might very well observe large sustained oscillations or even failure of the algorithm to drive the system to the desired set point. Such incidents have been reported by Song et al. (1984) and Gustafsson (1984). An adaptive controller based on the minimization of a quadratic index that includes a penalty on control deviation from the steady state value (Clarke and Gawthrop, 1975) can reduce oscillations, but requires accurate knowledge of the steady state control values; otherwise it results in offset. In addition, a high value of penalty results in sluggish response, which is unnecessary if the on-line identified model is good. This last problem is shared by STC's based on the minimization of a quadratic index that includes penalty in the control velocity (e.g. Clarke and Gawthrop, 1975). Velocity penalization may actually result in increased oscillations as well. Pole/zero placement adaptive controllers are faced with similar problems, in addition to the need to solve Bezoutian equations with the associated problems of ill-conditioning (Kumar and Moore, 1983). An alternative for some nonlinear systems is the use of nonlinear adaptive controllers. Promising algorithms have been proposed by Golden and Ydstie (1985) and by Gustafsson (1984), but these require that the nonlinearities be known *a priori* and be static, conditions seldom fulfilled in practice.

This paper presents a simple linear-model-based self-tuning controller that, when applied to nonlinear systems, leads to a closed-loop behavior free of the above mentioned problems. The key idea is the incorporation of two parts in the control law: a minimum variance part that depends directly on the estimated parameters of an assumed linear model, and a "caution" part that is independent of these estimated parameters. If the parameter estimates are poor the caution part is dominant and drives the system slowly but safely toward the desired steady state. When the estimates improve to the point that the plant/model mismatch is negligible, the minimum variance part becomes dominant and rapidly brings the system to the target steady state without large oscillations.

Cautious Self-Tuning Controller Structure

The well-known minimum-variance STC of Åström and Wittenmark (1973) assumes that the process can be described by the discrete-time single-input/single-output model

$$y(t+k+1) = A(q^{-1})y(t+k) + B(q^{-1})u(t) + s \quad (1)$$

$$A(q^{-1}) = a_0 + a_1q^{-1} + \cdots + a_nq^{-n} \quad (2)$$

$$B(q^{-1}) = b_0 + b_1q^{-1} + \cdots + b_mq^{-m} \quad (3)$$

where t denotes time in number of sampling intervals; y and u are the sampled system output and the control variable, respectively; $A(q^{-1})$ and $B(q^{-1})$ are polynomials in the backward shift operator $q^{-1}[q^{-1}f(t) = f(t-1)]$; s is a load parameter; and k is the system dead time in number of sampling intervals. The model parameters a_i, b_i, s are assumed unknown and may be slowly time-varying. It is however required that the dead time, k, and upper bounds to the system orders n and m be known *a priori*.

The MV STC attempts to minimize the performance measure

$$J[u(t)] = E\{[y(t+k+1) - y_s]^2|t\} \quad \text{(stochastic case)}$$
$$= [y(t+k+1) - y_s]^2 \quad \text{(deterministic case)} \quad (4)$$

where y_s is the desired set point and E is the expectation operator. To solve the minimization problem, the predictor form of Eq. 1 is first obtained by sequentially substituting for $y(t+k)$, $y(t+k-1), \ldots, y(t+1)$ using Eq. 1 shifted by the appropriate number of sampling intervals:

$$y(t+k+1) = C(q^{-1})y(t) + D(q^{-1})u(t) + r \quad (5)$$

where

$$C(q^{-1}) = c_0 + c_1q^{-1} + \cdots + c_nq^{-n} \quad (6)$$

$$D(q^{-1}) = d_0 + d_1q^{-1} + \cdots + d_{m+k}q^{-(m+k)}; \quad d_0 = b_0 \quad (7)$$

or in vector notation

$$y(t+k+1) = \underline{\theta}^T\underline{x}(t) = \underline{\theta}_1^T\underline{x}_1(t) + d_0u(t) \quad (8)$$

where

$$\underline{\theta}_1^T = [c_0, c_1, \ldots, c_n, d_1, d_2, \ldots, d_{m+k}, r] \quad (9)$$

$$\underline{x}_1^T(t) = [y(t), \ldots, y(t-n), u(t-1), \ldots, u(t-m-k), 1] \quad (10)$$

$$\underline{\theta}^T = [\underline{\theta}_1^T, d_0] \quad (11)$$

$$\underline{x}^T(t) = [\underline{x}_1^T(t), u(t)] \quad (12)$$

Substitution of Eq. 8 into Eq. 4 directly gives that the minimum variance control law is

$$u_{MV}^*(t) = \frac{1}{d_0}[y_s - \underline{x}_1^T(t)\underline{\theta}_1] \quad (13)$$

This achieves $y(t+k+1) = y_s$ and, from a pole-placement point of view, cancels some of the closed-loop poles with the open-loop zeroes and sets the rest of them to the origin. Thus $u_{MV}^*(t)$ is a controller that maximizes speed of response.

For the STC, since the parameter vector is unknown, it is estimated via a recursive estimator and the current estimates $\hat{\underline{\theta}}(t)$ are used in Eq. 13 in the place of the true parameters. The implemented control law is

$$u_{MV}(t) = \frac{1}{\hat{d}_0(t)}[y_s - \underline{x}_1^T(t)\hat{\underline{\theta}}_1(t)] \quad (14)$$

where ^ denotes parameter estimate.

The above-described MV STC has recently been proved to be globally stable when applied to a linear system (Ydstie and Sargent, 1982; Cordero and Mayne, 1981; Goodwin et al., 1980). It is a superior controller for linear or almost linear processes but is unreliable for highly nonlinear processes and can lead to failures of various types, ranging from unacceptably oscillatory regulation around the set point to instability. They are due to the high sensitivity of the MV control law on the estimated model, which can be very poor when the nonlinearities become dominant.

In what follows a new adaptive control scheme is presented that greatly improves on the robustness of the MV STC. Like the above-described STC it uses an on-line identified linear model, Eq. 8. However, recognizing that the true process is probably nonlinear, it is only partially dependent on this linear model, its influence disappearing whenever it fails to match the process behavior.

The cautious STC (CSTC) is based on a performance index of the type

$$J_2[u(t)] = [\hat{y}(t+k+1|t) - y_s]^2 + q(t)[u(t) - u_c(t)]^2 \quad (15)$$

where the caution term $u_c(t)$ is a conservative control law insensitive to or independent of the estimated model, and it is able to drive the system close to the desired set point. Possible choices of $u_c(t)$ are discussed in the next section. The future estimate $\hat{y}(t+k+1|t)$ is given by

$$\hat{y}(t+k+1|t) = \underline{x}^T(t)\hat{\underline{\theta}}(t) \quad (16)$$

where the estimate $\hat{\underline{\theta}}(t)$ is obtained using a recursive estimator. A UDU factorization (Bierman, 1977) of a recursive least-squares estimator with variable forgetting factor (Ydstie and Sargent, 1982; Fortescue et al., 1981) is well suited for identifying the linear part of a nonlinear process. The nonnegative weight $q(t)$ is not constant but an increasing function of the plant/model mismatch, becoming zero when the on-line identified model describes the process perfectly.

Minimization of performance measure J_2 with respect to $u(t)$ results in

$$\hat{y}(t+k+1|t) - y_s + \frac{q(t)}{\hat{d}_0(t)}[u(t) - u_c(t)] = 0 \quad (17)$$

or, in view of Eqs. 16 and 14, the CSTC control law is

$$u(t) = h(t)u_c(t) + [1 - h(t)]u_{MV}(t) \quad (18)$$

where

$$h(t) = \frac{q(t)}{\hat{d}_0^2(t) + q(t)} \quad (19)$$

Since $0 \leq h(t) \leq 1$, the controller of Eq. 18 is a convex combination of the terms $u_c(t)$ and $u_{MV}(t)$. If the estimated parameter vector $\hat{\theta}(t)$ provides an accurate description of the system, then the weight $q(t)$ should be taken close to zero and $u_{MV}(t)$ would be the dominant term. In the case of large plant/model mismatch a large value of $q(t)$ is appropriate, which would make the caution term $u_c(t)$ dominant and weigh out the erroneous $u_{MV}(t)$ contribution. A method for relating $q(t)$ to plant/model mismatch is given next.

Eq. 17 may be written alternatively as

$$e(t+k+1) - \epsilon(t+k+1)|t) = -\frac{q(t)}{\hat{d}_0(t)}[u(t) - u_c(t)] \quad (20)$$

where

$$e(t) = y(t) - y_s \quad (21)$$

is the output error, and

$$\epsilon(t|t-k-1) = y(t) - \hat{y}(t|t-k-1) \quad (22)$$

is the output prediction error given information up to time $t - k - 1$. If a local linear model is valid the output error is

$$e(t+k+1) = y(t+k+1) - y_s = d_0[u(t) - u^*_{MV}(t)] \quad (23)$$

where $u^*_{MV}(t)$ is given by Eq. 13 (i.e., it is based on the true parameter values). Substitution of Eq. 23 into Eq. 20 gives

$$u(t) = h'(t)u_c(t) + [1 - h'(t)]u^*_{MV}(t) + \frac{\hat{d}_0(t)h'(t)}{q(t)}\epsilon(t+k+1|t) \quad (24)$$

where

$$h'(t) = \frac{q(t)}{d_0\hat{d}_0(t) + q(t)} \quad (25)$$

The last term of Eq. 24 represents the erroneous part of the control law due to the inaccuracy of the on-line identified model. It is reasonable to limit that part to a fraction δ of the caution part, or

$$\left|\frac{\hat{d}_0(t)h'(t)}{q(t)}\epsilon(t+k+1|t)\right| = \delta|h'(t)u_c(t)| \quad (26)$$

from which follows

$$q(t) = \frac{|\hat{d}_0(t)|}{\delta|u_c(t)|}\left|\epsilon(t+k+1|t)\right| \quad (27)$$

The above equation expresses $q(t)$ as a function of the output prediction error $\epsilon(t+k+1|t)$. Since this error is unknown at time t, it is subsituted by a filtered $\epsilon(t|t-k-1)$. The filtering, by not allowing large sudden drops in $q(t)$, protects against the case where, at some instant, parameter errors combine so as to produce a small $\epsilon(t|t-k-1)$ even though the model is poor; in addition, by restricting the rate of change of $q(t)$, it smooths out the control law. Thus the weight $q(t)$ is finally chosen as

$$q(t) = \frac{|\hat{d}_0(t)|}{\delta|u_c(t)|}\nu(t) \quad (28)$$

where the filtered error $\nu(t)$ is

$$\nu(t) = p\nu(t-1) + (1-p)|\epsilon(t|t-k-1)|; \quad p \epsilon [0, 1) \quad (29)$$

The corresponding $h(t)$ is given by Eq. 19 as

$$h(t) = \frac{\nu(t)}{\delta|\hat{d}_0(t)u_c(t)| + \nu(t)} \quad (30)$$

One final note is in order. To assure that in Eq. 24 $h'(t) \cdot \epsilon [0, 1]$, we set $h(t) = 1$ [which implies $q(t) \rightarrow \infty$ and hence $h'(t) = 1$] whenever $\hat{d}_0(t)$ has the wrong sign. Thus *a priori* knowledge of the sign of d_0 is required.

With the above choice of $h(t)$ a large plant/model mismatch drives $h(t)$ toward 1 and hence the CSTC control law, Eq. 18, toward $u_c(t)$. As estimates improve and the mismatch decreases, $h(t)$ decreases and the control approaches $u_{MV}(t)$, as desired. However, it should be mentioned that the MV STC runs into stability problems if applied to systems with zeroes outside the open unit circle at the target steady state. Therefore the CSTC should also not be used in such cases.

The Caution Term

In this section the CSTC design is completed by presenting two possible choices for the caution term $u_c(t)$.

The first and simplest possibility is

$$u_{c1}(t) = u_s + \xi(t) \quad (31)$$

where u_s is a constant approximating the control variable value at the target steady state. Such an approximate value is usually known from operating experience or from a steady state model available from the design stage. Low-level white noise excitation $\xi(t)$ is added to aid the parameter estimator. Otherwise identifiability problems that arise for a constant or constant parameter control law (Åström and Wittenmark, 1973) would be encountered when h is high. The excitation component of $u(t)$ will of course decrease as the estimated model improves and h drops.

If a constant weight q were used, an inaccurate u_s would result in steady state offset. However, in the CSTC case q, and hence the contribution of $u_{c1}(t)$ in $u(t)$, decreases as the parameter estimates improve, leading ultimately to zero or insignificant offset.

The second possible choice for $u_c(t)$ requires *a priori* availability of an approximate, perhaps very rough, dynamic linear model. If such a model is not available one can operate the process open-loop, but with excitation added to the control variable, about a steady state, for a limited period of time. A least-squares estimator can then use the data collected to estimate the parameters of the locally valid linear model, Eq. 1.

Once a rough model has been obtained, the caution term can be designed as a very conservative internal model controller (Garcia and Morari, 1982; Brosilow and Tong, 1978). Let the transfer function between the control and measured variables be

$$\frac{\bar{y}(z)}{\bar{u}(z)} = G(z) = G_+(z)G_-(z) \tag{32}$$

where $\bar{}$ denotes z-transform and $G_+(z)$ includes all the noninvertible elements of $G(z)$, that is, the dead time and the zeroes on or outside the unit circle. Then the internal model controller is

$$\bar{u}_{IMC}(z) = -\frac{G_-^{-1}(z)F(z)}{1 - G_+(z)F(z)}\bar{e}(z) \tag{33}$$

where the filter $F(z)$ is included to increase robustness and is given by

$$F(z) = \frac{1-f}{1-fz-1}; \quad f\epsilon[0, 1) \tag{34}$$

For example, if $B(q^{-1})$ has no zeroes outside the open unit circle, the internal model controller for Eq. 1 is

$$B(q^{-1})[1 - fq^{-1} - (1-f)q^{-k-1}]u_{IMC}(t) = -(1-f)[1 - q^{-1}A(q^{-1})]e(t) \tag{35}$$

If the filter pole f is chosen very close to 1, u_{IMC} is very conservative (and sluggish) but will lead the system to the desired steady state, even if the model is poor. This is exactly what is required for the caution term, and therefore we set

$$u_{c2}(t) = u_{IMC}(t) + \xi(t) \tag{36}$$

It should be remarked here that a high value is recommended for f, even if the local model used to design $u_{IMC}(t)$ adequately represents the current state of process. Since plants are generally nonlinear, disturbance or set point changes may significantly alter the locally valid linear model. The sluggishness of $u_{IMC}(t)$, and hence $u_{c2}(t)$, is not a serious drawback since the $u_{MV}(t)$ component will speed up the system response as soon as the parameter estimates, aided by the excitation of $u_{c2}(t)$, improve.

Table 1. Parameter Values

Estimator Parameters
- $\gamma = 1 \times 10^{-5}$
- $\sigma = 1 \times 10^{-5}$
- $\lambda_{min} = 0.1$

Scaling Parameters
- $C_{ref} = 0.5$ lb mol/ft^3 = 8.01 kmol/m^3
- $Q_{cref} = 0.2$ ft^3/s = 5.664×10^{-3} m^3/s

CSTC Parameters
- $\delta = 0.20$
- $p = 0.95$
- $E\{\xi^2(t)\} = 0.01$
- u_s (when $y_s = 0.05$) = 0.5 (correct value = 0.573)
- u_s (when $y_s = 0.10$) = 0.9 (correct value = 1.02)
- u_s (when $y_s = 0.95$) = 1.4 (correct value = 1.57)
- $f = 0.999$

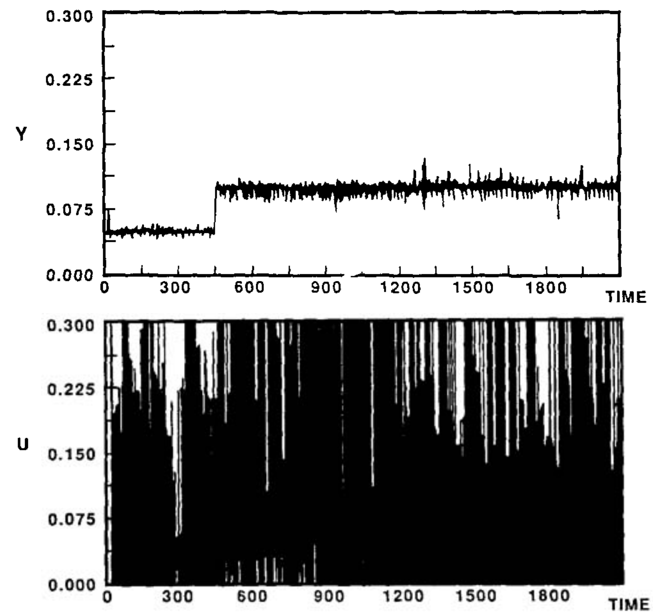

Figure 1. Reactor controlled by MV STC.
Top, concentration $y(t)$
Bottom, coolant flow rate $(u)t$

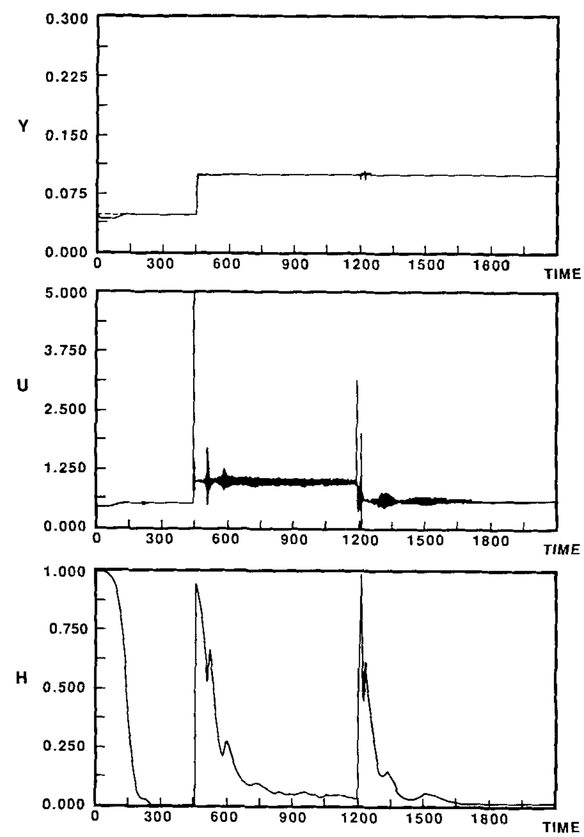

Figure 2. Reactor controlled by CSTC with caution term $u_{c1}(t)$.
Top, concentration $y(t)$; --- set point
Middle, coolant flow rate $(u)t$
Bottom, weight $h(t)$

Caution term $u_{c2}(t)$ is more difficult to obtain than $u_{c1}(t)$, but has the advantage that it guarantees complete elimination of steady state offset even for h different than zero.

Example

The CSTC performance is demonstrated on a stirred-tank reactor model given by Ahlgren and Stevens (1971). The reactor mass and energy balances are

$$\frac{dC}{dt} = \frac{Q}{V} C_i - \frac{Q}{V} C - Ke^{-E/RT} C \tag{37}$$

$$\frac{dT}{dt} = \frac{Q}{V} T_i - \frac{Q}{V} T + \frac{(-\Delta H) K e^{-E/RT}}{\rho c_p} C - \frac{UA}{\rho c_p V} \frac{2\rho_c c_{pc} Q_c (T - T_c)}{2\rho_c c_{pc} Q_c + UA} \tag{38}$$

The model parameter values are given in Ahlgren and Stevens (1971).

The adaptive controllers tested employ the UDU factorization of a recursive least-squares with variable forgetting factor given in the Appendix with parameters as in Table 1. To reduce numerical problems in the estimator, the input and output variables are scaled so that the components of $\underline{x}(t - k - 1)$ (Eqs. 13 and A1) do not differ widely in order of magnitude. Thus

$$y(t) = C(t)/C_{ref}; \quad u(t) = Q_c(t)/Q_{cref} \tag{39}$$

where the reference values are in Table 1. The table also includes all the CSTC parameters.

Figures 1–5 examine controller performance for small changes in the set point and the load C_i. At time $t = 450$ (sampling intervals of 20 s) the set point was changed from $y_s = 0.05$ to 0.10 and at $t = 1{,}200$ the inlet concentration was decreased by 10%. Figure 1 presents the performance of the MV STC. It is seen that it is fairly successful in following the set point and in rejecting the load change. However, the control variable behavior is unacceptably oscillatory. Figure 2 depicts the CSTC performance with caution term $u_{c1}(t)$. For both set points the *a priori* known u_s is initially 10% below the correct value and, since the inlet concentration drop is unmeasured, the u_s value is not altered at $t = 1{,}200$, resulting in an increase of the u_s error to 58%. It is seen that both the output and input are considerably improved. The last plot in Figure 2 is of $h(t)$. After the set point and disturbance changes, it increases, reflecting the increase in plant/model mismatch, and subsequently it drops as the parameter estimates improve. For comparison purposes Figure 3 shows the system response for $u(t) = u_s$. Since the u_s values are incorrect, offset results. The CSTC response for caution part $u_{c2}(t)$ is given in Figure 4. An approximate model, Eq. 1, is first obtained by estimating parameters while running the process about the initial steady state with white noise excitation. At $t = 50$ the CSTC starts and controls the reactor successfully. To be safe the filter constaat was assigned the high value $f = 0.999$. For this value the IMC response is extremely sluggish, as seen in Figure 5.

Figures 6–10 show controller performance for the large set point change from 0.95 to 0.10. The MV STC, as seen in Figure 6, completely fails to lead the process to the new set point. Instead, the coolant flow rate is stuck at the lower bound. In con-

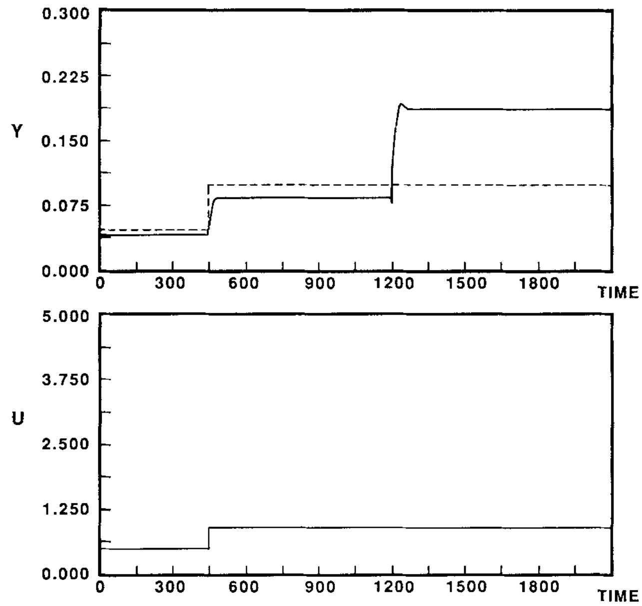

Figure 3. Reactor run open-loop, $u(t) = u_s$.
Top, concentration $y(t)$; --- set point
Bottom, coolant flow rate $u(t)$

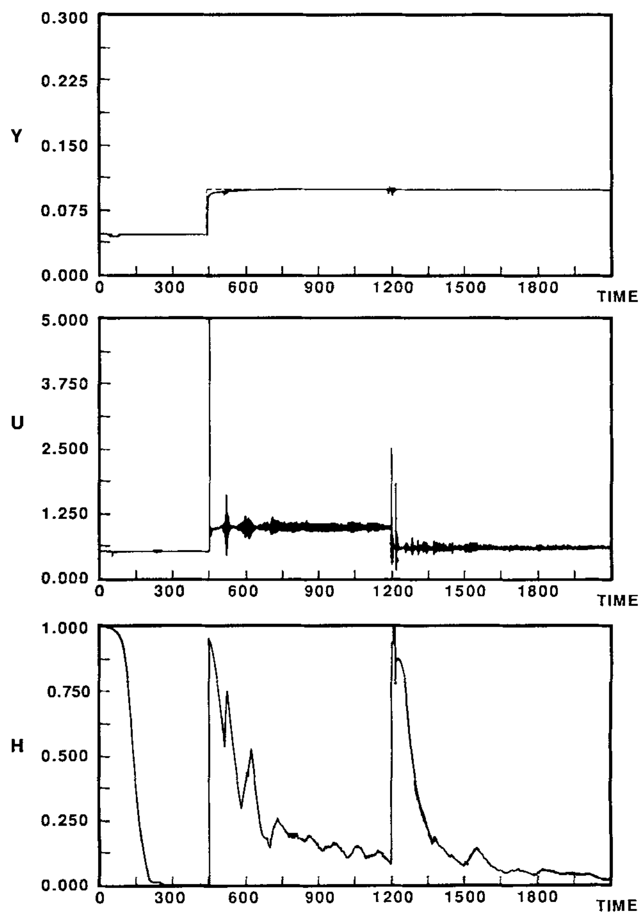

Figure 4. Reactor controlled by CSTC with caution term $u_{c2}(t)$.
Top, concentration $y(t)$
Middle, coolant flow rate $u(t)$
Bottom, weight $h(t)$

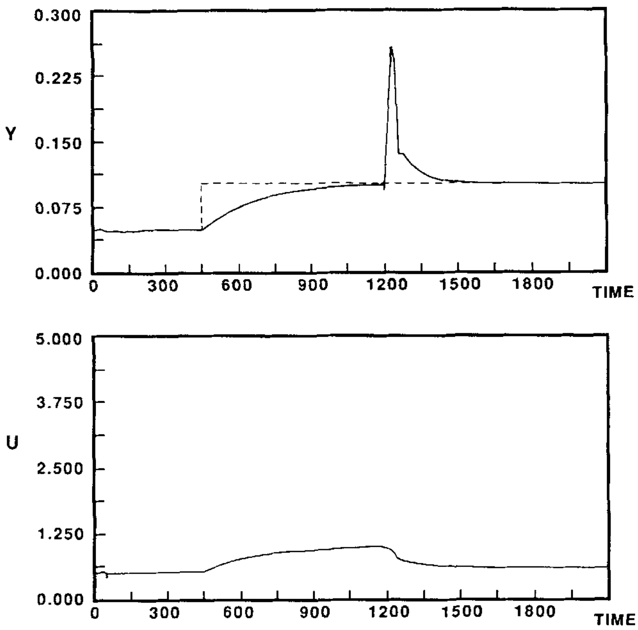

Figure 5. Reactor controlled by $u_{IMC}(t)$.
Top, concentration $y(t)$; --- set point
Bottom, coolant flow rate $u(t)$

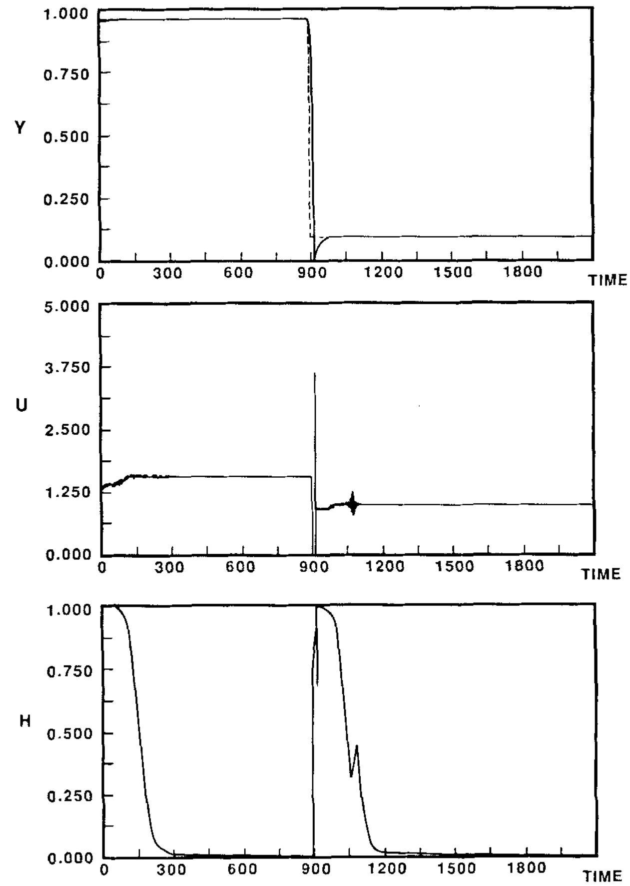

Figure 7. Reactor controlled by CSTC with caution term $u_{c1}(t)$.
Top, concentration $y(t)$; --- set point
Middle, coolant flow rate $u(t)$
Bottom, weight $h(t)$

trast, the CSTC with caution $u_{c1}(t)$ succeeds, Figure 7, even though the *a priori* specified u_s is off by 10%. It is worth noting that the reactor has steady state multiplicity, and for control $u(t) = u_{c1}(t)$ the process winds up at the wrong steady state, Figure 8. The successful performance of the CSTC with $u_{c2}(t)$ is shown in Figure 9. Figure 10 depicts the sluggish response of $u(t) = u_{c2}(t)$. Finally, it should be mentioned that the CSTC responds quite well to step changes in any of the disturbances. For example, Figure 11 depicts the CSTC response to the

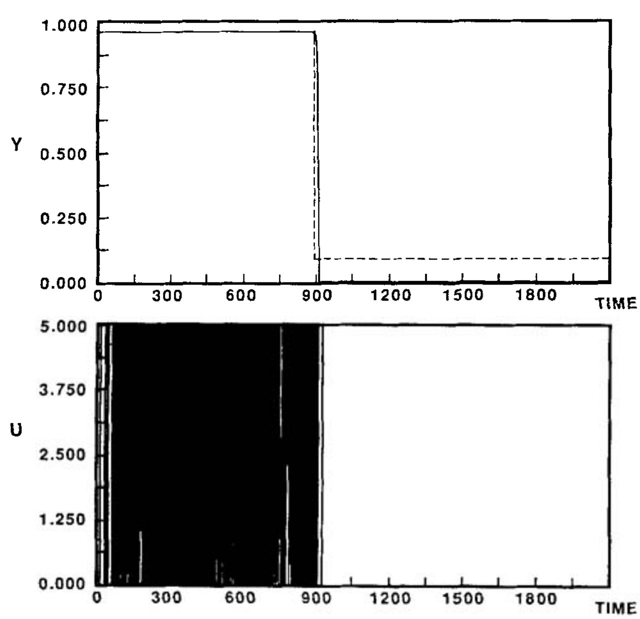

Figure 6. Reactor controlled by MV STC.
Top, concentration $y(t)$; --- set point
Bottom, coolant flow rate $u(t)$

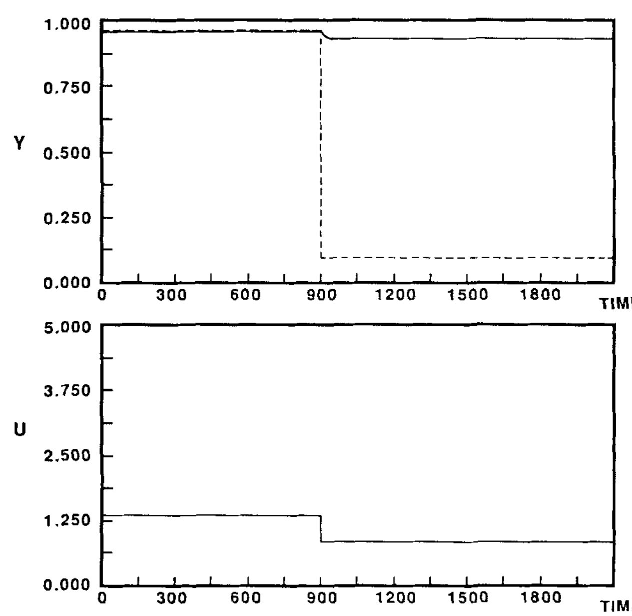

Figure 8. Reactor run open-loop, $u(t) = u_s$.
Top, concentration $y(t)$; --- set point
Bottom, coolant flow rate $u(t)$

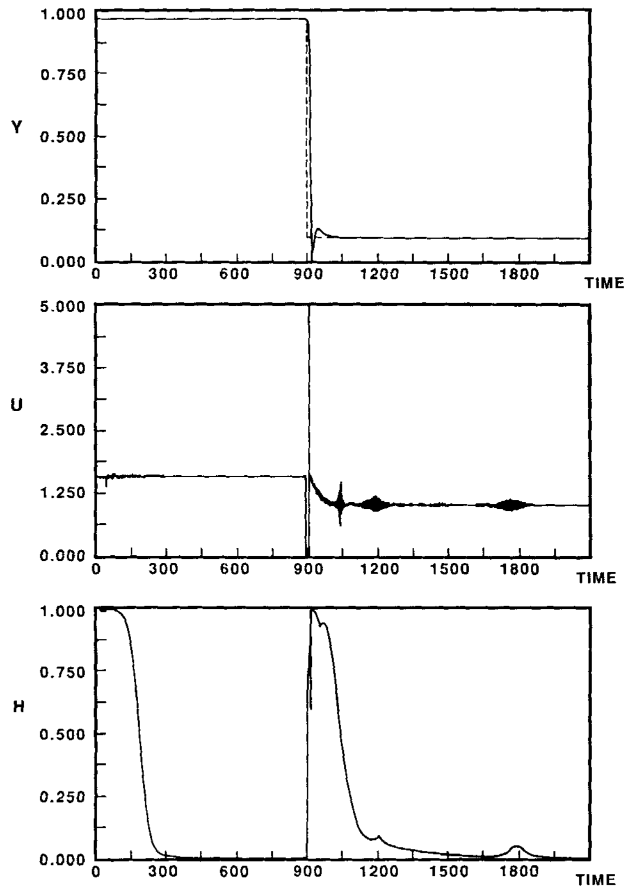

Figure 9. Reactor controlled by CSTC with caution term $u_{c2}(t)$.
Top, concentration $y(t)$; --- set point
Middle, coolant flow rate $u(t)$
Bottom, weight $h(t)$

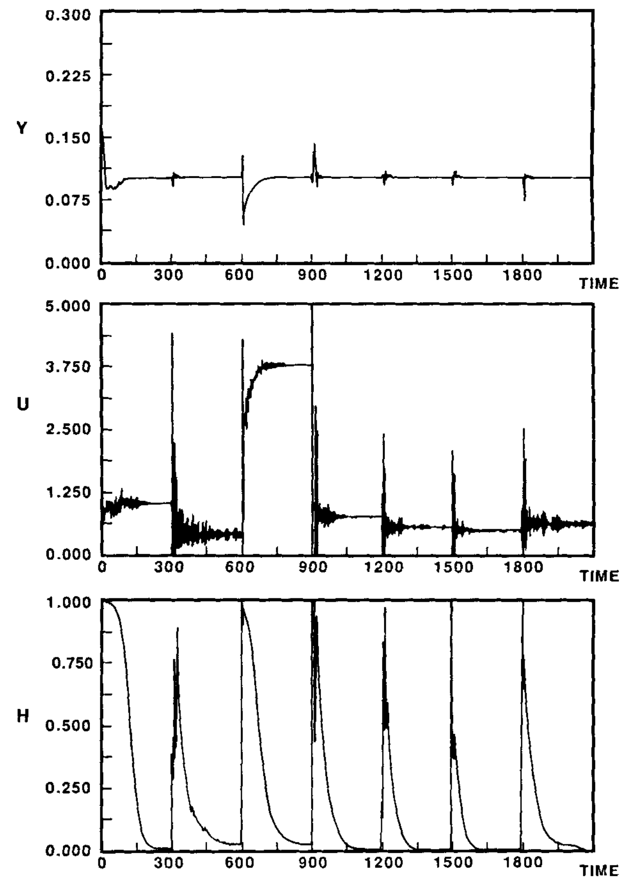

Figure 11. Reactor controlled by CSTC with caution term $u_{c1}(t)$.
Top, concentration $y(t)$
Middle, coolant flow rate $u(t)$
Bottom, weight $h(t)$

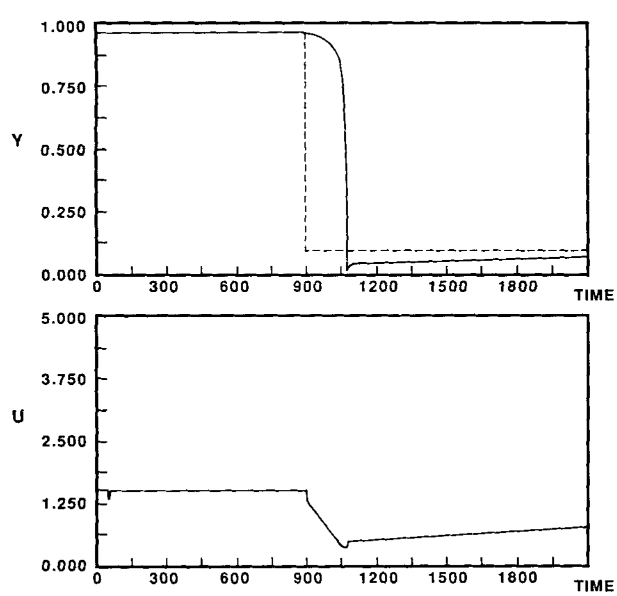

Figure 10. Reactor controlled by $u_{IMC}(t)$.
Top, concentration $y(t)$; --- set point
Bottom, coolant flow rate $u(t)$

changes:

(i) 20% drop in C_i at $t = 300$
(ii) 50% increase in C_i at $t = 600$
(iii) 5% drop in T_i (in K) at $t = 900$
(iv) 8% drop in T_c at $t = 1{,}200$
(v) 11% increase in Q at $t = 1{,}500$
(vi) 9% drop in UA at $t = 1{,}800$

It should be pointed out that the CSTC, by slowing down the system and adding excitation, aids the parameter estimator. As a result the model identified by the CSTC differs from that identified by the MV STC even though both have been tuned identically, and when h approaches 0 the CSTC control law performs considerably better than the MV STC law.

Conclusions and Significance

The cautious self-tuning controller (CSTC) links a conservative robust control law and the minimum variance control law, which is a controller that gives very fast closed-loop dynamics. The CSTC adjusts the weighing between its two parts, giving conservative control when the parameter estimates are poor, but speeding up system response as the estimates improve. Although it requires a low amount of modeling information, it performs well for both servo-tracking and regulation, even in the presence of strong nonlinearities. These features make it an attractive alternative for the control of chemical plants, which are nonlinear and for which reliable dynamic models do not generally exist.

Acknowledgment

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Notation

$A(q^{-1})$ = polynomial in backward shift operator
a_i = coefficients of $A(q^{-1})$
A = heat exchange area
$B(q^{-1})$ = polynomial in backward shift operator
b_i = coefficients of $B(q^{-1})$
$C(q^{-1})$ = polynomial in backward shift operator
c_i = coefficients of $C(q^{-1})$
C = concentration
C_i = inlet concentration
c_p = reactor specific heat capacity
c_{pc} = coolant specific heat capacity
C_{ref} = reference concentration
$D(q^{-1})$ = polynomial in backward shift operator
d_i = coefficients of $D(q^{-1})$
E = activation energy
$e(t)$ = output error
$F(z)$ = IMC filter transfer function
f = IMC filter pole
$G(z)$ = model transfer function
$G_+(z)$ = factor of G containing noninvertible elements
$G_-(z)$ = factor of G containing invertible elements
$h(t)$ = control law weight
ΔH = enthalpy change of reaction
K = reaction rate preexponential factor
p = estimation error filter parameter
Q = inlet flow rate
Q_c = coolant flow rate
Q_{cref} = reference coolant flow rate
$q(t)$ = performance index weight
R = gas constant
r = load parameter
s = load parameter
T = reactor temperature
T_c = coolant temperature
T_i = inlet temperature
t = time in sampling interval units
U = heat transfer coefficient
u = control variable
u_c = caution term
u_{IMC} = IMC control law
u^*_{MV} = minimum variance control law
u_{MV} = minimum variance approximation using estimated parameters
u_s = steady state control value
V = reactor volume
$\underline{x}(t)$ = regression vector containing known inputs and outputs
$\underline{x}_1(t)$ = regression vector containing known inputs and outputs
$y(t)$ = measured variable
y_s = steady state output value
z = z-domain independent variable

Greek letters

δ = control law parameter
$\epsilon(t + k + 1|t)$ = output prediction error
$\underline{\theta}(t)$ = model parameter vector
$\underline{\theta}_1(t)$ = model parameter vector
k = time delay in number of sampling intervals
λ = forgetting factor
λ_{min} = minimum value of λ
$\nu(t)$ = filtered output prediction error
$\xi(t)$ = white noise sequence
ρ = reactor density
ρ_c = coolant density
σ = forgetting factor parameter

Superscripts

$\hat{}$ = estimated variable
T = transposition
$-$ = z-transformed variable

Appendix: Recursive Least-Squares Estimator with Variable Forgetting Factor

The UDU factorization (Bierman, 1977) of the Fortescue et al. (1981) recursive least-squares estimator for model

$$y(t) = \underline{x}^T(t - k - 1)\underline{\theta} \qquad \text{(A1)}$$

is as follows.

Initialization

$\hat{\underline{\theta}}(0) = \underline{\theta}_0$
(If no *a priori* estimate is available the typical choice is $\underline{\theta}_0^T = [0, \ldots, 0, 1]$)
$U(0) = [\underline{u}_1(0), \underline{u}_2(0), \ldots, \underline{u}_n(0)] = I$
$d_i(0) = 1/\gamma; \quad i = 1, \ldots, n$

Recursive steps

$\underline{f} = U^T(t-1)\underline{x}(t-k-1)$
$\underline{f}^T = [f_1, f_2, \ldots, f_n]$
$v_i = d_i(t-1)f_i; \quad i = 1, \ldots, n$
$w = 1 + \Sigma_{i=1}^n v_i f_i$
$\epsilon = y(t) - \underline{x}^T(t-k-1)\hat{\underline{\theta}}(t-1)$ (prediction error)
$\bar{\lambda} = 1 - \epsilon^2/[(1+w)\sigma]$
$\lambda = \max[\bar{\lambda}, \lambda_{min})$ (forgetting factor)
$\beta_1 = 1 + v_1 f_1$
$\beta_i = \beta_{i-1} + v_i f_i; \quad i = 1, \ldots, n$
$d_1(t) = d_1(t-1)/(\beta_1\lambda)$
$d_i(t) = d_i(t-1)\beta_{i-1}/(\beta_i\lambda); \quad i = 2, \ldots, n$
$\ell_i = -f_i/\beta_{i-1}; \quad i = 2, \ldots, n$
$\underline{k}_2^T = [v_1, 0, \ldots, 0]$
$\underline{k}_{i+1} = \underline{k}_i + v_i\underline{u}_i(t-1); \quad i = 2, \ldots, n$
$\underline{k} = \underline{k}_{n+1}/\beta_n$
$\hat{\underline{\theta}}(t) = \hat{\underline{\theta}}(t-1) + \underline{k}\epsilon$ (new estimate)
$\underline{u}_1(t) = \underline{u}_1(t-1)$
$\underline{u}_i(t) = \underline{u}_i(t-1) + \ell_i\underline{k}_i; \quad i = 2, \ldots, n$

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